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The condition that the algorithm terminates on the first iteration is that exactly one computer picks the value 1, while the others pick values other than 1. The probability of picking the value 1 for one computer is , and the probability of picking values other than 1 is . The probability that one computer chooses 1, and the others (n-1) computers choose non-1 values is . Because the graph is a clique, it is symmetric with respect to all computers. Therefore, the probability that exactly one computer choose 1 and others (n-1) do not choose one is:  
 (proven)

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The probability that no leader is picked is the complement of the probability found in (a):   
. By the k-iteration, the probability that no leader is picked is:

*  (answer)

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Let’s call X as the general runtime of the algorithm.

We have: 

* = 
* 
* 
* 
* 
*  (geometric sum)
* . At 

The expected run time is (answer)

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From part (a), we have: => 

From part (b), the probability of no leaders being chosen by k-iteration is:

, and c can be any value. Choose c = 2

* 
* 
* 
* 
*  (since ). Since  is constant, denote it as -C
* 

As we can see, when n grows, then the term  will become much smaller. The chance of k longer than  becomes greatly diminished as n grows. Therefore, the algorithm runs in  time with high probability

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Because the two sets  and are symmetric, the proof needs to be done for one case. Let’s pick the set  and call the diameter of  as . The probability of being  is:



Given an arbitrary path in , the probability of this path being included in  is . Therefore, the probability of  being is:



Because diameter is the largest distance possible between any two vertices, we can deduce that



Applying the union bound to the above probability, the identity becomes:



Since the number of paths in each set is the number of vertices pair, which is 

* 

By definition, we have: , But because 

*  . Multiply both sides by, we have
* 

Because the term can outgrow much faster than, as increases, grows indefinitely and will increase the failing probability. In other words, the probability of obtaining a set with diameter  will decrease.

* Using the same algorithm in this case does not guarantee success with high probability